PRINCIPIUM GEOMETRICUM

Geometric Persistence, Orthogonal Phase Planes, and the Dimensional-Emergence Test

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$$(\Sigma^2, \theta_U, \theta_P) \rightarrow (M^{3+1}, g^{PG}_{\mu\nu})$$

ABSTRACT

This paper reformulates geometric persistence in the Principium Geometricum (PG) research program around a precise structural hypothesis: an observed effective geometry with three spatial dimensions and one time dimension may arise from a two-dimensional spatial substrate carrying two coupled phase sectors. The arrow in the equation above is treated as a problem to solve, not as an established result. A pre-relativistic candidate action is introduced for the phases. In the symmetric regime it contains a gapless common mode and a gapped relative mode. The phase sectors are then coupled to the collective toroidal coordinates (R, a) through an energy Hessian, producing a concrete generalized eigenvalue problem for hybrid shape-phase oscillations.

The main conceptual result is a no-shortcut criterion: an additional normal mode is not, by itself, an additional spatial dimension. Dimensional emergence requires a family of low-energy states with locality, translation, an independently variable coordinate, and a universal propagation law. The relativistic complex-field model is retained as an effective benchmark, while Lorentz symmetry and gravitation remain targets to be recovered rather than assumptions of the fundamental model.

Keywords: geometric persistence; phase coherence; toroidal field configurations; dimensional emergence; emergent Lorentz symmetry; effective geometry.

1. Structural Hypothesis

The central PG hypothesis considered here is

$$3D + 1T \;<\!-^{PG}\; 2D + T^2$$

The notation T^2 does not yet denote two physical times. It labels two cyclic degrees of freedom, θ_U and θ_P, provisionally associated with update and propagation. Their orthogonality is initially kinematic: they are independent coordinates in the internal configuration space. Physical orthogonality and the emergence of a new spatial direction must follow from dynamics.

The original oscillatory expression is retained as historical input:

$$T(t) = \alpha_U \sin(\Omega t) + A_P \cos(\Omega t)$$
$$A_P = \ell_P^{\,2}, \quad \alpha_U = A_P k_e$$

Because the coefficients are not dimensionally compatible in SI units, a normalized geometric variable is required:

$$T(t) = A_P[\eta_U \sin(\Omega t) + \cos(\Omega t)]$$
$$\eta_U = k_e/k_*$$

The reference scale k_* and the physical meaning of T must be derived. Since A_P contains G, this ansatz cannot by itself derive Newton's constant.

2. Effective Field-Theory Benchmark

A Lorentz-invariant complex scalar field remains useful as an effective reference theory:

$$\Phi = f\, e^{i\theta}$$
$$L_{eff} = K|\partial_t \Phi|^2/(2c^2) - K|\nabla \Phi|^2/2 - U(|\Phi|)$$

It provides a conserved global phase charge and a positive field-energy functional. For a stable isolated solution, its rest energy gives $M_{eff} = E_0/c^2$. In a homogeneous collective truncation:

$$E_{bulk}(V) = u_o V + Q^2/(2\chi V)$$
$$V_* = |Q|/\sqrt{(2\chi u_o)}, \quad E_* = |Q|\sqrt{(2u_o/\chi)}$$

These relations establish a conditional volume minimum, not the existence or full stability of a soliton. The reference Lagrangian already contains c and Lorentz symmetry; it benchmarks the desired macroscopic behavior but cannot explain the origin of that symmetry.

3. Two-Phase Fundamental Candidate

Let Σ^2 be a two-dimensional spatial substrate and λ an auxiliary evolution parameter. A minimal candidate action is

$$S_o = \int d\lambda\, d^2x\, L_{PG}$$

$$L_{PG} = (I_U/2)(\partial_\lambda \theta_U)^2 + (I_P/2)(\partial_\lambda \theta_P)^2 - (H_U/2)|\nabla_2 \theta_U|^2 - (H_P/2)|\nabla_2 \theta_P|^2 - J[1 - \cos(\theta_U - \theta_P)] - V_{tor}$$

No spacetime metric or gravitational field is inserted. The coefficients are dynamical parameters to be derived or measured. Positivity requires $I_A > 0$ and $H_A > 0$; the locked equilibrium below requires $J > 0$.

$$\theta_+ = (\theta_U + \theta_P)/2$$
$$\theta_- = (\theta_U - \theta_P)/2$$

In the symmetric regime $I_U = I_P = I$ and $H_U = H_P = H$, linearization around $\theta_- = 0$ gives

$$\omega_+^2 = (H/I)k^2$$
$$\omega_-^2 = (H/I)k^2 + 2J/I$$

The common mode is gapless and the relative mode is gapped. The characteristic speed $v_0 = \sqrt{(H/I)}$ is derived from phase-action coefficients, but identifying it with the universal measured speed c requires universality across all low-energy sectors.

4. Toroidal Energy and Shape-Phase Coupling

For a circular torus with major radius R, minor radius a, and $R > a > 0$,

$$V = 2\pi^2 R a^2, \quad S = 4\pi^2 R a$$

For winding number n, constant interior amplitude f_0, and a sharp toroidal region, the exact idealized circulation energy is

$$E_{circ} = 2\pi^2 K f_o^2 n^2 (R - \sqrt{(R^2 - a^2)})$$

A reduced energy retaining bulk, charge, surface, and circulation terms is

$$E_o(R,a) = u_o V + Q^2/(2\chi V) + \sigma S + E_{circ}(R,a)$$

To couple shape and phases without hiding the assumption, write

$$E_{PG} = E_o(R,a) + J(R,a)[1 - \cos(\theta_U - \theta_P)]$$

At an equilibrium $q_* = (R_*, a_*, \theta_{U*}, \theta_{P*})$, define

$$K_{ij} = [\partial^2 E_{PG}/(\partial q_i \partial q_j)]_{q^*}$$
$$\det(K - \omega^2 M) = 0$$

At the exactly locked point $\theta_U - \theta_P = 0$, the simplest coupling above has vanishing linear shape-phase mixing because the mixed derivatives are proportional to $\sin(\theta_U - \theta_P)$. Hybridization therefore requires a displaced phase equilibrium, velocity-dependent coupling, or additional symmetry-allowed terms. This is a concrete restriction, not a failure that can be removed by terminology.

5. The Dimensional-Emergence Test

The vector $(R, a, \theta_U, \theta_P)$ describes a finite-dimensional configuration space. Its normal modes are oscillations, not coordinates of physical space. A relative phase can qualify as an emergent spatial coordinate only if the theory produces all of the following:

1. Independent addressability. Low-energy states can be localized at distinct values of the candidate coordinate.
2. Locality. Interactions decay or remain local with respect to separation in that coordinate.
3. Translation structure. A continuous approximate symmetry generates momentum conjugate to the coordinate.
4. Extended spectrum. The mode supplies a continuum or a sufficiently dense tower, not one isolated oscillator.
5. Universal propagation. Different stable excitations recover the same effective causal cone.
6. Geometric coupling. Matter and clocks respond to one effective metric within experimental limits.

additional normal mode ≠ additional spatial dimension

A viable construction may require a phase fiber over every point of Σ^2, topological defects whose relative phase becomes an extended coordinate, or a large family of coupled toroidal cells. The present action establishes the phase sectors but not this final step.

6. From Relative-Phase Networks to an Operational Third Spatial Direction

The dimensional-emergence criteria can be tested without introducing a continuous coordinate z at the fundamental level. Let $G = (V, E)$ be a network of N phase states over each point of Σ^2, and let $\varphi_i(x, \lambda)$ be the relative-phase excitation at node i. A quadratic benchmark action is

$$S_{net} = \int d\lambda\, d^2x \left[\Sigma_i (I(\partial_\lambda \varphi_i)^2/2 - H|\nabla_2 \varphi_i|^2/2) - (J/2)\Sigma_{ij} A_{ij}(\varphi_i - \varphi_j)^2 \right]$$

The adjacency matrix A_{ij} must be selected by the underlying toroidal dynamics. If L_G is the graph Laplacian and $L_G u^{(m)} = \lambda_m u^{(m)}$, the normal modes obey

$$\omega_m^2(k_\perp) = (H/I)|k_\perp|^2 + (J/I)\lambda_m$$

The index m is spectral and does not initially represent a spatial momentum. An operational k_z exists only if a low-energy family of eigenmodes can be ordered continuously, wave packets propagate locally through the network, and an approximate translation generator acts on that family.

6.1 The uniform-chain benchmark

For a periodic nearest-neighbor chain, used only as a benchmark,

$$\lambda(q) = 4 \sin^2(q/2), \quad q = 2\pi m/N$$

If an operational spacing a_z can be derived from correlations or propagation time, define $k_z = q/a_z$. At long wavelength,

$$\omega^2 \simeq v_\perp^2 |k_\perp|^2 + v_z^2 k_z^2$$
$$v_\perp^2 = H/I, \quad v_z^2 = Ja_z^2/I$$

An isotropic effective spatial sector requires $v_z = v_\perp$, while relativistic universality further requires the common limiting speed to converge to c for every low-energy species. A chain already carries one-dimensional adjacency, so it demonstrates the mechanism but does not derive the topology. PG must explain why its microscopic couplings select a chain-like low-energy graph.

6.2 Spectral-dimension criterion

The network can be tested without assigning coordinates by its return probability

$$P(s) = N^{-1} \operatorname{Tr} \exp(-sL_G)$$
$$d_s(s) = -2 \, d \ln P(s) / d \ln s$$

A one-dimensional internal geometry has P(s) proportional to $s^{-1/2}$ and therefore $d_s \simeq 1$ over a nontrivial scaling window. Combined with the two-dimensional substrate, the effective spectral dimension should approach three. This behavior must persist as N grows and must not be an artifact of a preselected chain.

6.3 Decision test

The phase network supports an operational third direction only if it simultaneously yields:

1. a dynamically selected adjacency with a stable $d_s \simeq 1$ regime;
2. local wave-packet propagation and a measurable network distance;
3. a continuous low-energy label that behaves as an independent k_z;
4. an isotropic dispersion with $v_z = v_\perp$; and
5. the same limiting speed and effective metric for distinct matter modes.

Failure of any condition distinguishes an internal phase network from a physical spatial direction. Passing all of them would not yet derive gravity, but it would supply the missing mechanism behind the proposed passage from 2D + T^2 to an operational 3D space.

7. From Effective Propagation to Geometry

Perturbations about a background solution define principal kinetic operators. If all low-energy species share the same hyperbolic operator, its characteristic cone may be encoded in an effective inverse metric:

$$G^{\mu\nu}{}_{AB}(q_*) \, \partial_\mu \partial_\nu \delta\varphi^B + \ldots = 0$$

A single metric $g^{PG}_{\mu\nu}$ exists only if the species and polarization dependence factorizes appropriately. Otherwise the model predicts multiple propagation cones or composition-dependent clock behavior and is constrained by Lorentz-invariance tests.

$$S_0 \to S_{eff} \to g^{PG}_{\mu\nu} \to \text{Einstein-Hilbert limit}$$

Every arrow requires a separate derivation. The existence of a gapless scalar mode does not imply a massless spin-2 sector. Recovering general relativity requires the correct tensor polarizations, gauge redundancy or its effective analogue, universal matter coupling, nonlinear self-interaction, and verified weak-field and radiative limits.

8. Decisive Calculations

1. Choose a bounded microscopic form of V_{tor} and derive its coefficients independently.
2. Derive the phase-network adjacency from toroidal interactions instead of selecting a chain by hand.
3. Compute the graph spectrum, return probability, and spectral dimension as the network size grows.
4. Test whether wave packets define a local network distance and an independent low-energy k_z.
5. Solve the equilibrium equations and full shape-phase Hessian, including nonaxisymmetric perturbations.
6. Test $v_z = v_\perp$ and derive characteristic cones for multiple excitations.
7. Search separately for a massless tensor sector and derive its coupling to total energy.

The first calculation is already discriminating. For the minimal locked cosine coupling, linear shape-phase hybridization vanishes at the symmetric equilibrium. A nonzero hybrid mode therefore requires additional dynamics that must be stated explicitly and tested for stability.

9. Status of Claims

Derived within stated approximations. Conserved phase charge in the effective complex-field benchmark; conditional bulk-volume minimum; field-derived collective inertia; exact idealized toroidal circulation integral; gapless and gapped phase modes of the symmetric two-plane action; generalized Hessian eigenvalue problem; absence of linear shape-phase mixing for the minimal locked coupling.

Current hypotheses. Two independent phase sectors are fundamental; their collective dynamics can generate an effective third spatial direction and time; a universal effective metric can arise; a spin-2 gravitational sector may emerge at long wavelengths.

Not yet derived. A microscopic origin for all coefficients; stable finite-energy three-dimensional matter; the third spatial coordinate; Lorentz symmetry; universal clock behavior; Newton's constant; Einstein dynamics; gravitational-wave polarizations; a distinctive confirmed PG prediction.

10. Conclusion

The two-phase reformulation gives the PG program a sharper logical structure. It separates an internal two-sector phase space from observed spacetime and converts the proposed equality between them into a sequence of dynamical tests. The toroidal energy and complex-field calculations remain useful conditional results, while the pre-relativistic action provides a place to investigate whether a universal propagation cone can emerge.

The strongest result is also a limitation: the minimal two-phase action yields common and relative oscillations, but neither oscillation is automatically a new spatial direction. The next stage is therefore not to assume 3D + 1T, but to construct and test the locality, spectrum, translation symmetry, and universal coupling needed for it. If those conditions fail, the hypothesis 3D + 1T <- 2D + T^2 is constrained in a precise and informative way.

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